

**An Alternative String Theory in Twistor Space
for N=4 Super-Yang-Mills**

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In this letter, an alternative string theory in twistor space is proposed for describing perturbative N=4 super-Yang-Mills theory. Like the recent proposal of Witten, this string theory uses twistor worldsheet variables and has manifest spacetime superconformal invariance. However, in this proposal, tree-level super-Yang-Mills amplitudes come from open string tree amplitudes as opposed to coming from D-instanton contributions.

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In a recent paper [1], Witten has shown that the simple form of maximal helicity violating amplitudes of Yang-Mills theory has a natural generalization to the non-maximal helicity violating amplitudes. He also constructed a topological B-model from twistor worldsheet variables and argued that D-instanton contributions in this model reproduce the perturbative super-Yang-Mills amplitudes. For details on this model and the twistor approach to super-Yang-Mills, see the review and references in [1].

The formula for D-instanton contributions of degree d to n -gluon tree-level amplitudes is [1][2]

$$B(\lambda_r, \bar{\lambda}_r) = \int d^{2d+2}a \, d^{2d+2}b \, d^{4d+4}\gamma \prod_{r=1}^n \int d\sigma_r \, (\text{vol}(GL(2)))^{-1} \quad (1)$$

$$\prod_{r=1}^{n-1} (\sigma_r - \sigma_{r+1})^{-1} (\sigma_n - \sigma_1)^{-1} \prod_{r=1}^n \delta\left(\frac{\lambda_r^2}{\lambda_r^1} - \frac{\lambda^2(\sigma_r)}{\lambda^1(\sigma_r)}\right) \exp(i\bar{\lambda}_r^{\dot{\alpha}} \lambda_r^1 \frac{\mu_{\dot{\alpha}}(\sigma_r)}{\lambda^1(\sigma_r)})$$

$$\text{Tr}[\phi_1(\frac{\psi^A(\sigma_1)}{\lambda^1(\sigma_1)}) \phi_2(\frac{\psi^A(\sigma_2)}{\lambda^1(\sigma_2)}) \dots \phi_n(\frac{\psi^A(\sigma_n)}{\lambda^1(\sigma_n)})]$$

where $P_r^{\alpha\dot{\alpha}} = \lambda_r^\alpha \bar{\lambda}_r^{\dot{\alpha}}$ is the momentum of the r^{th} state,

$$\lambda^\alpha(\sigma) = \sum_{k=0}^d a_k^\alpha \sigma^k, \quad \mu^{\dot{\alpha}}(\sigma) = \sum_{k=0}^d b_k^{\dot{\alpha}} \sigma^k, \quad \psi^A(\sigma) = \sum_{k=0}^d \gamma_k^A \sigma^k,$$

$\phi_r(\psi^A)$ is the N=4 superfield whose lowest component is the positive helicity gluon and whose top component is the negative helicity gluon, and the $(\text{vol}(GL(2)))^{-1}$ factor can be used to remove one of the a integrals and three of the σ integrals.

For maximal helicity violating amplitudes (i.e. $n - 2$ positive helicity gluons and 2 negative helicity gluons), the above formula when $d = 1$ has been shown to give the correct n -point amplitude. For non-maximal helicity violating amplitudes, it has been suggested that this formula may also give the correct n -point amplitude where one has $n - d - 1$ positive helicity gluons and $d + 1$ negative helicity gluons. Although there is a possibility that the formula of (1) needs to be modified for non-maximal amplitudes by contributions from instantons of lower degree, it has been recently verified that no such modifications are necessary when $d = 2$ and $n = 5$ [2]. It will be assumed below that the formula of (1) correctly reproduces the super-Yang-Mills tree amplitudes for any d and n .

In this letter, a new string theory in twistor space is proposed which reproduces the formula of (1) using ordinary open string tree amplitudes as opposed to D-instanton contributions. This string theory shares many aspects in common with the original idea

of Nair in [3]. The worldsheet matter variables in this string theory consist of a left and right-moving set of super-twistor variables,

$$Z_L^I = (\lambda_L^\alpha, \mu_L^{\dot{\alpha}}, \psi_L^A), \quad Z_R^I = (\lambda_R^\alpha, \mu_R^{\dot{\alpha}}, \psi_R^A) \quad (2)$$

for $\alpha, \dot{\alpha} = 1$ to 2 and $A = 1$ to 4, a left and right-moving set of conjugate super-twistor variables,

$$Y_{LI} = (\bar{\mu}_{L\alpha}, \bar{\lambda}_{L\dot{\alpha}}, \bar{\psi}_{LA}), \quad Y_{RI} = (\bar{\mu}_{R\alpha}, \bar{\lambda}_{R\dot{\alpha}}, \bar{\psi}_{RA}), \quad (3)$$

and a left and right-moving current algebra,

$$j_L^C, \quad j_R^C \quad (4)$$

where C is Lie-algebra valued and j_L^C and j_R^C satisfy the usual OPE's of a current algebra, i.e.

$$j_L^C(y)j_L^D(z) \rightarrow \frac{f^{CDE}j_L^E(z)}{y-z} + \frac{kg^{CD}}{(y-z)^2}, \quad j_R^C(\bar{y})j_R^D(\bar{z}) \rightarrow \frac{f^{CDE}j_R^E(\bar{z})}{\bar{y}-\bar{z}} + \frac{kg^{CD}}{(\bar{y}-\bar{z})^2}. \quad (5)$$

The current algebra can be constructed from free fermions, a Wess-Zumino-Witten model, or any other model.

The worldsheet action is

$$S = \int d^2z (Y_{LI} \nabla_R Z_L^I + Y_{RI} \nabla_L Z_R^I) + S_G \quad (6)$$

where S_G is the worldsheet action for the current algebra and (∇_R, ∇_L) contains a worldsheet $\text{GL}(1)$ connection for which Z_L^I and Z_R^I have charge $+1$, and Y_{LI} and Y_{RI} have charge -1 .

Quantizing this worldsheet action gives rise to left and right-moving Virasoro ghosts, (b_L, c_L) and (b_R, c_R) , as well as left and right-moving $\text{GL}(1)$ ghosts, (u_L, v_L) and (u_R, v_R) . The untwisted left-moving stress tensor is

$$T_0 = Y_{LI} \partial_L Z_L^I + T_G + b_L \partial_L c_L + \partial_L (b_L c_L) + u_L \partial_L v_L \quad (7)$$

where T_G is the left-moving stress tensor for the current algebra, and the left-moving $\text{GL}(1)$ current is

$$J = Y_{LI} Z_L^I. \quad (8)$$

To have vanishing conformal anomaly, the current algebra must be chosen such that the central charge contribution from T_G is 28. Note that there is no $GL(1)$ anomaly because of cancellation between bosons and fermions in J .

The open string theory is defined using the conditions

$$Z_L^I = Z_R^I, \quad Y_{LI} = Y_{RI}, \quad j_L^C = j_R^C, \quad c_L = c_R, \quad b_L = b_R, \quad v_L = v_R, \quad u_L = u_R \quad (9)$$

on the open string boundary. Unlike a usual open string theory where Lie algebra indices come from Chan-Paton factors, the Lie algebra indices in this open string theory come from a current algebra.

The physical integrated and unintegrated open string vertex operator for the super-Yang-Mills states is

$$V = \int dz j^C(z) \Phi_C(Z(z)), \quad U = c(z) j^C(z) \Phi_C(Z(z)). \quad (10)$$

The superfields $\Phi_C(Z)$ are similar to those defined in [1], namely for a super-Yang-Mills state with momentum $P_r^{\alpha\dot{\alpha}} = \lambda_r^\alpha \bar{\lambda}_r^{\dot{\alpha}}$,

$$\Phi_C(Z(z_r)) = \delta\left(\frac{\lambda_r^2}{\lambda_r^1} - \frac{\lambda^2(z_r)}{\lambda^1(z_r)}\right) \exp\left(i \bar{\lambda}_r^{\dot{\alpha}} \lambda_r^1 \frac{\mu^{\dot{\alpha}}(z_r)}{\lambda^1(z_r)}\right) \phi_C\left(\frac{\psi^A(z_r)}{\lambda^1(z_r)}\right) \quad (11)$$

where $\phi_C(\psi^A)$ is the same $N=4$ superfield as in (1). Note that $\Phi_C(Z)$ is $GL(1)$ -neutral and has zero conformal weight.

Tree-level open string scattering amplitudes are computed in the usual manner from the disk correlation function

$$A = \langle U_1(z_1) U_2(z_2) U_3(z_3) \int dz_4 V_4(z_4) \dots \int dz_n V_n(z_n) \rangle \quad (12)$$

where different twistings of the stress tensor are used to compute different helicity violating amplitudes. For amplitudes involving $(n-d-1)$ positive helicity gluons and $d+1$ negative helicity gluons, the twisted stress tensor is defined as

$$T_d = T_0 + \frac{d}{2} \partial J \quad (13)$$

where T_0 and J are defined in (7) and (8). Note that T_d has no conformal anomaly since J has no $GL(1)$ anomaly.

So after twisting, Z^I has conformal weight $-\frac{d}{2}$ and Y_I has conformal weight $\frac{d+2}{2}$. This means that the disk correlation function of (12) involves an integration over the $4d+4$

bosonic and $4d+4$ fermionic zero modes of Z^I , except for the one bosonic zero mode which can be removed using the worldsheet $GL(1)$ gauge invariance. Performing the correlation function for the current algebra gives the contribution²

$$Tr[\phi_1 \dots \phi_n] \prod_{r=1}^{n-1} (z_r - z_{r+1})^{-1} (z_n - z_1)^{-1}, \quad (15)$$

and the (b, c) correlation function gives the factor $(z_1 - z_2)(z_2 - z_3)(z_3 - z_1)$.

So one obtains the formula

$$A = \int d^{2d+2}a \, d^{2d+2}b \, d^{4d+4}\gamma \int dz_1 \dots \int dz_n (Vol(GL(2)))^{-1} \quad (16)$$

$$\prod_{r=1}^{n-1} (z_r - z_{r+1})^{-1} (z_n - z_1)^{-1} \prod_{r=1}^n \delta\left(\frac{\lambda_r^2}{\lambda_r^1} - \frac{\lambda^2(z_r)}{\lambda^1(z_r)}\right) \exp(i\bar{\lambda}_r^{\dot{\alpha}} \lambda_r^1 \frac{\mu_{\dot{\alpha}}(z_r)}{\lambda^1(z_r)})$$

$$Tr[\phi_1(\frac{\psi^A(z_1)}{\lambda^1(z_1)}) \phi_2(\frac{\psi^A(z_2)}{\lambda^1(z_2)}) \dots \phi_n(\frac{\psi^A(z_n)}{\lambda^1(z_n)})]$$

where

$$\lambda^\alpha(z) = \sum_{k=0}^d a_k^\alpha z^k, \quad \mu^{\dot{\alpha}}(z) = \sum_{k=0}^d b_k^{\dot{\alpha}} z^k, \quad \psi^A(z) = \sum_{k=0}^d \gamma_k^A z^k,$$

$(a_k^\alpha, b_k^{\dot{\alpha}}, \gamma_k^A)$ are the zero modes of Z^I on a disk, and the $SL(2)$ part of $GL(2)$ can be used to fix three of the z_r integrals and reproduce the (b, c) correlation function. This formula clearly agrees with the formula of (1) for the D-instanton amplitude where the σ variable from the D1-string worldvolume has been replaced with the z variable from the open string boundary.

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² As was pointed out to me by Edward Witten, one also gets multitrace contributions such as

$$Tr[\phi_1 \dots \phi_m] Tr[\phi_{m+1} \dots \phi_n] \left(\prod_{r=1}^{m-1} (z_r - z_{r+1})^{-1} (z_m - z_1)^{-1} \right) \left(\prod_{s=m+1}^{n-1} (z_s - z_{s+1})^{-1} (z_n - z_{m+1})^{-1} \right) \quad (14)$$

coming from other contractions of the current algebra. These multitrace contributions are also present in the amplitudes coming from D-instantons in [1] and in the proposal of Nair in [3].

References

- [1] E. Witten, *Perturbative Gauge Theory as a String Theory in Twistor Space*, hep-th/0312171.
- [2] R. Roiban, M. Spradlin and A. Volovich, *A Googly Amplitude from the B-Model in Twistor Space*, hep-th/0402016.
- [3] V.P. Nair, *A Current Algebra for some Gauge Theory Amplitudes*, Phys. Lett. B214 (1988) 215.